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Deformation of conifold and intersecting branes

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Abstract: We study the relation between intersecting NS5-branes whose intersection is smoothed out and the deformed conifold in terms of the supergravity solution. We solve the condition of preserved supersymmetry on a metric inspired by the deformed conifold metric and obtain a solution of the NS5-branes which is delocalized except for one of the overall transverse directions. The solution has consistent properties with other configurations obtained by string dualities.

Keywords: p-branes, String Duality, Superstring Vacua.

Contents

1. Introduction

A system of parallel D3-branes at a conifold [1, 2, 3, 4] has been discussed from the viewpoint of AdS/CFT correspondence [5] (and for a review [6]). From the properties of the conifold, we can identify the field theory on the N D3-branes with $\mathcal{N} = 1$ gauge group $\mathrm{SU}(N) \times \mathrm{SU}(N)$ theory with a quartic superpotential in the infrared [2].

By using T-dualities, the system is mapped to configurations of D-branes and intersecting NS5-branes over a $1 + 3$ dimensional world-volume [7, 8]. In the brane picture, one can intuitively read out the gauge group and the spectrum of the microscopic theory on the D-branes.

The T-duality relations between intersecting NS5-branes and the conifold play an important role in order to map the system of D3-branes at the conifold singularity to the brane configurations.

The relation between the NS5-branes and the conifold is discussed in terms of supergravity solutions by performing the T-duality along one of the overall directions of the NS5-branes [7]. The T-duality maps the field theory on the D3-branes at the conifold singularity onto D4-branes which are stretched between both sides of NS5-branes and extend along the compactified direction. Such configurations are called elliptic models [9]. The duality relation is generalized to other conifold types and various types of NS5-branes [8, 10, 11, 12].

There is another T-duality relation between the intersecting NS5-branes and the conifold [13]. The NS5-branes are mapped to the conifold by performing two T-dualities along one of the relative transverse directions of each NS5-brane. By taking these T-dualities, one has D5-branes which fill the compact brane box [14, 15]. It is

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discussed that the intersecting point of NS5-branes must be resolved to obtain the field theory on the D5-branes which has suitable gauge group and superpotential [11]. The intersecting NS5-branes are also described by a single NS5-brane wrapping the curve which supports a non trivial S^1 that D5-branes can end. Such NS5-brane is called NS5-brane with a "diamond". In the conifold picture, the conifold singularity is resolved.

In the absence of D-branes, local mirror symmetry is proposed between generalized and orbifolded conifolds [11]. The mirror transformation is equivalent to the T-duality on the supersymmetric toroidal 3-cycle which Calabi-Yau manifolds and their mirror manifolds equip [16]. In the conifolds picture, the T-duality is equivalent to the combination of above two types of T-dualities. Performing the mirror transformation to the blownup conifold, one find that the deformed conifold is mapped to the NS5-brane with the diamond by the T-duality along one of the overall transverse directions.

Our purpose in the present paper is to explore the relation between the NS5-branes with the diamond and the deformed conifold in terms of supergravity solutions. The guideline to obtain the deformed conifold metric is discussed in [17] and the explicit metric is presented in [18]. We start with a metric inspired by the deformed conifold metric. Solving the condition of the preserved supersymmetry, after some replacements of line elements [7] and the T-duality, we will have a solution of the NS5-branes with a diamond which is delocalized except for one of the overall transverse directions. As a result, we find that the size of the diamond relates to the displacement of end points of a D4-brane on a NS5-brane. We confirm it by using string dualities.

This paper is organized as follows. In section 2, we present a short summary on the duality relations between metics of the NS5-branes and the conifold [7], and discuss the deformation of the conifold algebraic geometry [11]. We also explicitly give the deformed conifold metric [17, 18]. In section 3, we construct the NS5-branes with the diamond metric and show that the metric is also obtained from the intersecting NS5-branes metric by some coordinate transformation. In section 4, we make clear the meaning of the size of the diamond by using some string dualities. Section 5 is devoted to conclusion and discussion.

2. NS5-branes and conifold

2.1 Duality relations

Let us start with intersecting NS5 and NS5'-branes whose world-volume directions are

$$
\begin{array}{c|l}
\mathrm{NS5} & (0 \quad\quad\quad 5\ 6\ 7\ 8\ \ 9) \\
\mathrm{NS5}' & (0 \quad 3\ 4 \quad\quad 7\ 8\ 9)\,.
\end{array}
\tag{2.1}
$$

We consider the intersecting NS5-branes metric smeared except for the x^1-direction. The metric relates to the conifold metric by the T-duality along the x^2-direction [7].

The conifold is topologically a cone over a 5-dimensional base manifold $S^2 \times S^3$. To see the relation between NS5-branes and the conifold, it is useful to consider the geometry of the base manifold as U(1) fibration over a base $S^2 \times S^2$. In the intersecting NS5-branes background, we have a similar geometry which is S^1 over $\mathbb{R}^2 \times \mathbb{R}^2$. Here S^1 is the x^2-direction which is compactified to take the T-duality, and $\mathbb{R}^2 \times \mathbb{R}^2$ is the (x^3, x^4) and (x^5, x^6) directions which are planar. Therefore we must "compactify" the directions in order to fit the topology to the conifold. This is done by replacing the Mauer-Cartan 1-form of $\mathbb{R}^2 \times \mathbb{R}^2$ by of $S^2 \times S^2$, that is,

$$\begin{aligned} dx^{3,5} &\to \sin\theta_{1,2}\, d\phi_{1,2}\,, \\ dx^{4,6} &\to d\theta_{1,2}\,. \end{aligned} \tag{2.2}$$

After the T-duality, we have the conifold metric below up to coefficients with certain replacements, for example $x^2 \to \psi$ where ψ is the coordinate on the U(1) fiber. It is difficult to determine the coefficients from the intersecting NS5-branes metric because the metric is delocalized except for only one direction though the conifold metric localizes in the all directions.

The conifold metric is given in [17],

$$ds^2_{\rm conifold} = dr^2 + r^2 \left(\frac{1}{6} \sum_{i=1}^{2} (d\theta_i + \sin^2\theta_i\, d\phi_i^2) + \frac{1}{9}(d\psi + \cos\theta_1 d\phi_1 + \cos\theta_2 d\phi_2)^2 \right). \tag{2.3}$$

Here r is a radial coordinate in the conifold and relates to the x^1-direction in the intersecting NS5-branes picture.

The relation between the generalized and orbifolded conifold whose singularities are resolved is discussed in [11]. The simplest example of it is the T-duality or mirror symmetry between the deformed conifold and the blowup of conifold. The conifold has vanishing 2-cycle and 3-cycle at the origin where is an isolated singular point. In the intersecting NS5-branes picture, the singularity corresponds to a singular point where the NS5-brane intersects with the NS5'-brane. On the conifold the singularity can be resolved by deforming the conifold. In the NS5-brane picture, the deformation smooth out the intersection with a non-vanishing cycle.

The conifold is algebraically defined by $x\,y = u\,v$, where $(x, y, u, v) \in \mathbf{C}^4$. The deformed conifold is described by

$$xy = uv + \epsilon^2\,, \tag{2.4}$$

where the singularity is resolved by a 3-cycle with non-zero radius ϵ. By using the T-duality the deformed conifold maps to NS5-branes wrapping a curve which appears on $uv = 0$ [19],

$$xy = \epsilon^2\,. \tag{2.5}$$

In the conifold case, ϵ vanishes and the curve becomes $xy = 0$. The solution separates into $x = 0$ and $y = 0$. Each equation describes a location of the NS5-brane. Hence the conifold simply maps to intersecting NS5 and NS5'-branes after the T-duality. This fact agrees with the discussion using the metric. When the $\epsilon \neq 0$, the curve (2.5) is smooth and describes topologically a sphere of radius ϵ. We call the non-vanishing cycle which NS5-branes wrap as a "diamond" [11].

2.2 Metric of the deformed conifold

The metric of (2.4) is determined from the condition that the metric is Ricci flat and Kähler [17, 18],

$$ds^2 = \mathcal{F}' \, \mathrm{tr}(dW^\dagger dW) + \mathcal{F}'' \left|\mathrm{tr}(W^\dagger dW)\right|^2 . \tag{2.6}$$

Here W is a complex 2×2 matrix which satisfies the condition corresponding to (2.4),

$$\det W = -\frac{1}{2}\epsilon^2 . \tag{2.7}$$

Here we fix ϵ as a real parameter. We define a radial coordinate ρ^2 in $\mathbf{C}^4$ space as

$$\rho^2 \equiv \mathrm{tr}(W^\dagger W) . \tag{2.8}$$

$\mathcal{F} = \mathcal{F}(\rho^2)$ is a Kähler potential and $\mathcal{F}' \equiv d\mathcal{F}/d(\rho^2)$ is determined by the condition that the metric is Ricci flat as

$$\mathcal{F}' = \epsilon^{-2/3} \, K . \tag{2.9}$$

Here K is a function defined as

$$\begin{aligned} K(\tau) &\equiv \frac{(\sinh 2\tau - 2\tau)^{1/3}}{2^{1/3}\sinh\tau} , \\ \rho^2 &\equiv \epsilon^2 \cosh\tau . \end{aligned} \tag{2.10}$$

We now take W as

$$\begin{aligned} W &= L \, W_\epsilon \, R^\dagger , \\ W_\epsilon &= \begin{pmatrix} 0 & \frac{\sqrt{\rho^2+\epsilon^2}+\sqrt{\rho^2-\epsilon^2}}{2} \\ \frac{\sqrt{\rho^2+\epsilon^2}-\sqrt{\rho^2-\epsilon^2}}{2} & 0 \end{pmatrix} . \end{aligned} \tag{2.11}$$

The SU(2) matrix L, R are parametrized in terms of Euler angles,

$$\begin{pmatrix} \cos\frac{\theta_k}{2} \, e^{i\,(\psi_k+\phi_k)/2} & -\sin\frac{\theta_k}{2} \, e^{-i\,(\psi_k-\phi_k)/2} \\ \sin\frac{\theta_k}{2} \, e^{i\,(\psi_k-\phi_k)/2} & \cos\frac{\theta_k}{2} \, e^{-i\,(\psi_k+\phi_k)/2} \end{pmatrix} , \tag{2.12}$$

where $k = 1, 2$ for L, R respectively. The stability group of W_ϵ is a U(1) which fixes $\psi_1 + \psi_2 \to \psi$.

Eventually we have the deformed conifold metric,

$$ds^2 = K\epsilon^{\frac{4}{3}} \left(\frac{\sinh^3\tau}{3\,(\sinh 2\tau - 2\tau)} (d\tau^2 + ds_1^2) + \frac{\cosh\tau}{4} ds_2^2 + \frac{1}{4} ds_3^2 \right) , \tag{2.13}$$

where

$$ds_1^2 \equiv (d\psi + \cos\theta_1\, d\phi_1 + \cos\theta_2\, d\phi_2)^2\,, \tag{2.14}$$

$$ds_2^2 \equiv d\theta_1^2 + d\theta_2^2 + \sin^2\theta_1\, d\phi_1^2 + \sin^2\theta_2\, d\phi_2^2\,, \tag{2.15}$$

$$\begin{aligned} ds_3^2 \equiv 2\Big(&\sin\psi\,(d\phi_1\, d\theta_2\, \sin\theta_1 + d\phi_2\, d\theta_1\, \sin\theta_2) + \\ &+ \cos\psi\,(d\theta_1\, d\theta_2 - d\phi_1\, d\phi_2\, \sin\theta_1\, \sin\theta_2)\Big). \end{aligned} \tag{2.16}$$

The determinant of the deformed conifold metric is proportional to $\sinh^4\tau$ which vanishes if $\tau \to 0$, and the deformed conifold reduces to a lower-dimensional subspace. From eq. (2.10), the limit means that $\rho \to \epsilon$ where the stability group enhances to SU(2). At $\rho = \epsilon$, the geometry becomes $(\mathrm{SU}(2) \times \mathrm{SU}(2))/\,\mathrm{SU}(2) = S^3$. So the deformed conifold metric reduces to the S^3 surface metric.

If we take another limit,

$$\rho^2 = \epsilon^2 \cosh\tau \text{ fixed as } \tau, \frac{1}{\epsilon} \to \infty, \tag{2.17}$$

the deformed conifold metric (2.13) reduces to the conifold one (2.3) with a coordinate transformation $\rho^2 = (2/3)^{3/2} r^3$. Thus we confirm that when the size of 3-cycle ϵ vanishes the metric smoothly deforms to the conifold one.

The deformed conifold metric has the term (2.16) which does not exist in the conifold case (2.3). Since the deformed conifold is considered as the T-dual of the NS5-branes with a diamond, we expect that the additional term is closely related to an effect of the existence of diamond.

3. NS5-branes with a diamond metric

In this section, we consider the relation between the deformed conifold metric and the NS5-branes with the diamond metric which is smeared except for one of the overall transverse directions x^1 as in the previous section. The metric of the NS5-branes with the diamond relates to the deformed conifold metric after the T-duality. In the latter case, there is no gauge fields and dilaton background in the corresponding string theory. So we focus only on the metric of the T-dual of NS5-branes for a while and solve the condition for a preserved supersymmetry on the metric.

The deformed conifold metric (2.13) is fully localized. On the other hand, however, we are looking for a smeared metric. In this case, we assume that some replacements of the line elements would be again similar to the intersecting NS5-branes and conifold case,

$$\sin\theta_{1,2} d\phi_{1,2} \to dx_{3,5}\,, \tag{3.1}$$

$$d\theta_{1,2} \to dx_{4,6}\,, \tag{3.2}$$

and

$$d\psi \to dx_2\,. \tag{3.3}$$

We now would like to take the T-duality along the x^2-direction. Hence we need a U(1) isometry along the direction. However, we have functions depending on the x^2-coordinate which are $\sin x^2$ and $\cos x^2$ in the deformed conifold metric (2.13) in the above replacements. We assume that they become some constants a_1 and a_2 after delocalization. So eqs. (2.14), (2.15) and (2.16) become

$$
\begin{aligned}
ds_1^2 &= (dx^2 + B_1 dx^3 + B_2 dx^5)^2\,, \\
ds_2^2 &= \sum_{i=3}^{6} (dx^i)^2\,, \\
ds_3^2 &= 2\left(a_1\,(dx^3\,dx^6 + dx^5\,dx^4) + a_2\,(dx^4\,dx^6 - dx^3\,dx^5)\right).
\end{aligned}
\tag{3.4}
$$

Here B_1, B_2 are some functions which could not be determined by the above replacements even in the conifold case. They should be determined by solving the supersymmetry condition. We will specify the functions by an ansatz as we will see below.

Thus we consider the superstring compactification on the six-dimensional curved space which described by the following metric,

$$
\begin{aligned}
ds^2 &= g_{\mu\nu} dx^\mu dx^\nu \\
&= A(x^1)^2\,(dx^1)^2 + B(x^1)^2\,ds_1^2 + 2C(x^1)\,ds_2^2 + 2D(x^1)\,ds_3^2\,.
\end{aligned}
\tag{3.5}
$$

Here $A(x^1)$, $B(x^1)$, $C(x^1)$ and $D(x^1)$ are functions depending only on the x^1-coordinate. We have introduced a_1 and a_2 as arbitrary constants. If we rescale $\sqrt{a_1^2 + a_2^2} D(x^1)$ as $D(x^1)$ and $ds_3^2/\sqrt{a_1^2 + a_2^2}$ as ds_3^2, the constants a_1 and a_2 become $a_1/\sqrt{a_1^2 + a_2^2}$ and $a_2/\sqrt{a_1^2 + a_2^2}$ in ds_3^2 (3.4). Therefore the redefinition means that we simply set

$$
\begin{aligned}
a_1 &\to \sin\alpha\,, \\
a_2 &\to \cos\alpha\,,
\end{aligned}
\tag{3.6}
$$

where α is a constant.

Since we expect that above metric (3.5) becomes the solution of NS5-branes with a diamond after the T-duality along the x^2-direction, we make the following ansatz before solving the preserved supersymmetry condition:

(i) The both x^1 and x^2-directions are the overall transverse directions to the NS5-branes after the T-duality. Therefore the coefficients of $(dx^1)^2$ and $(dx^2)^2$ must be the same after that. Since the metric $g_{\mu\nu}$ and the T-dualized metric $j_{\mu\nu}$ are related each other by $j_{11} = g_{11}$ and $j_{22} = 1/g_{22}$ [20], it means that

$$
B(x^1) = A(x^1)^{-1} \tag{3.7}
$$

for the metric (3.5).

(ii) After the T-duality, non zero components of the NS-NS 2-field are $B_{23} = -g_{23}/g_{22} = -B_1$ and $B_{25} = -g_{25}/g_{22} = -B_2$. The NS5-brane charge is given by an integral of the 3-form field strength $H = dB$. We restrict that the NS5-branes charges are measured on the outside of the diamond. This is because the 'origin' of the deformed conifold is on the 3-cycle with non-zero radius. The deformed conifold metric is not defined inside the 3-cycle. The size of 3-cycle corresponds to the size of the diamond. Therefore we expect that the NS5-branes with the diamond metric inspired by the deformed conifold metric is also defined only on the outside of the diamond. Since we consider the case that all directions except for the one direction are smeared, we assume that the components of the field strength are constants. So the components of the NS-NS 2-form linearly depend on the coordinates, $B_{23} = \sum_{i=3}^{6} n_i\, x^i$ and $B_{25} = \sum_{i=3}^{6} m_i\, x^i$. We can set $n_3, m_5, m_3 = 0$ by using a gauge transformation. Thus we have

$$\begin{aligned} B_1 &= -(n_4\, x^4 + n_5\, x^5 + n_6\, x^6)\,, \\ B_2 &= -(m_4\, x^4 + m_6\, x^6)\,. \end{aligned} \tag{3.8}$$

Note that we still consider a single diamond, but we introduce n_i and m_i. This means that some individual NS5-branes wrap the same diamond.

Let us consider the IIA theory compactified on the 6-dimensional metric. Since there are no gauge fields and dilaton, we can trivially lift to the 11 dimensional theory. Therefore we equivalently consider unbroken supersymmetry in the 11-dimensional theory for simplicity.

The condition of preserved supersymmetry is that the supersymmetric variation with respect to the gravitino must vanish, namely $\delta\psi_\mu = 0$, in a vanishing gravitino background,

$$\begin{aligned} \delta\psi_\mu &= D_\mu \eta + \frac{1}{288}(\Gamma_{\mu\nu\rho\sigma\lambda} - 8 g_{\mu\nu}\Gamma_{\rho\sigma\lambda})F^{\nu\rho\sigma\lambda}\eta\,, \\ D_\mu\eta &= \partial_\mu\eta + \frac{1}{4}\omega_\mu{}^{\hat{a}\hat{b}}\Gamma_{\hat{a}\hat{b}}\eta\,, \end{aligned} \tag{3.9}$$

where F is a 4-form field strength and $\omega_\mu{}^{\hat{a}\hat{b}}$ is a spin connection. The Majorana spinor η is the supersymmetry parameter. The hatted indices refer to the $D = 11$ tangent space, $\eta_{\hat{a}\hat{b}} = \mathrm{diag}\,(-,+,\ldots,+)$, and $\Gamma_{\hat{a}}$ are the $D = 11$ Dirac matrices obeying

$$\{\Gamma_{\hat{a}}, \Gamma_{\hat{b}}\} = 2\eta_{\hat{a}\hat{b}}\,, \tag{3.10}$$

and

$$\Gamma_{\hat{a_1}\ldots\hat{a_n}} = \Gamma_{[\hat{a_1}}\ldots\Gamma_{\hat{a_n}]}\,. \tag{3.11}$$

Since there are no 4-form field strength in our case, the condition reduces to

$$D_\mu\eta = 0\,. \tag{3.12}$$

We choose normal coordinate basis $\theta^{\hat{a}}$ of the metric in 11-dimensions as

$$
\begin{aligned}
\theta^{\hat{1}} &= A\, dx^1\,, \\
\theta^{\hat{2}} &= A^{-1}\left(dx^2 - (n_4\, x^4 + n_5\, x^5 + n_6\, x^6)\, dx^3 - (m_4\, x^4 + m_6\, x^6)\, dx^5\right), \\
\theta^{\hat{3}} &= \sqrt{C-D}\,(-\sin\alpha\, dx^3 - \cos\alpha\, dx^4 + dx^6)\,, \\
\theta^{\hat{4}} &= \sqrt{C-D}\,(\cos\alpha\, dx^3 - \sin\alpha\, dx^4 + dx^5)\,, \\
\theta^{\hat{5}} &= \sqrt{C+D}\,(\sin\alpha\, dx^3 + \cos\alpha\, dx^4 + dx^6)\,, \\
\theta^{\hat{6}} &= \sqrt{C+D}\,(-\cos\alpha\, dx^3 + \sin\alpha\, dx^4 + dx^5)\,, \\
\theta^{\hat{i}} &= dx^i \qquad i = 0, 7, \ldots, 10\,.
\end{aligned}
\tag{3.13}
$$

The metric of the six-dimensional curved space (3.5) is given by $\sum_{i=1}^{6}(\theta^{\hat{i}})^2$. Here we assume that $C-D$ and $C+D$ are positive to make the metric be lorentzian.

Supposing that the supersymmetry parameter η depends only on the x^1, the non zero components of $D_\mu\eta$ relative to the metric (3.5) are

$$D_1\eta = \partial_1\eta\,, \tag{3.14}$$

$$D_2\eta = \frac{2\,f}{A^2}\,\eta\,, \tag{3.15}$$

$$D_3\eta = \left(-\frac{2\,(n_4x^4 + n_5x^5 + n_6x^6)}{A^2}f + \frac{1}{A}\,g_1 + \frac{1}{A}\,g_2\right)\eta\,, \tag{3.16}$$

$$D_4\eta = \left(\frac{1}{A}\,g_3 + \frac{1}{A}\,g_4\right)\eta\,, \tag{3.17}$$

$$D_5\eta = \left(-\frac{2\,(m_4x^4 + m_6x^6)}{A^2}f + \frac{1}{A}h_1\right)\eta\,, \tag{3.18}$$

$$D_6\eta = \frac{1}{A}h_2\,\eta\,, \tag{3.19}$$

where

$$f = \frac{A'}{A}\Gamma_{\hat{1}\hat{2}} + X_1\Gamma_{\hat{3}\hat{4}} + X_2\Gamma_{\hat{3}\hat{5}} + X_3\Gamma_{\hat{3}\hat{6}} - X_4\Gamma_{\hat{4}\hat{5}} - X_5\Gamma_{\hat{4}\hat{6}} - X_6\Gamma_{\hat{5}\hat{6}}\,, \tag{3.20}$$

$$g_1 = Y_1\Gamma_{\hat{3}\hat{1}} + Y_2\Gamma_{\hat{5}\hat{1}} + Z_1\Gamma_{\hat{4}\hat{2}} + Z_2\Gamma_{\hat{6}\hat{2}}\,, \tag{3.21}$$

$$g_2 = Y_3\Gamma_{\hat{4}\hat{1}} + Y_4\Gamma_{\hat{6}\hat{1}} + Z_3\Gamma_{\hat{3}\hat{2}} + Z_4\Gamma_{\hat{5}\hat{2}}\,, \tag{3.22}$$

$$g_3 = -Y_3\Gamma_{\hat{3}\hat{1}} - Y_4\Gamma_{\hat{5}\hat{1}} + Z_5\Gamma_{\hat{4}\hat{2}} + Z_6\Gamma_{\hat{6}\hat{2}}\,, \tag{3.23}$$

$$g_4 = Y_1\Gamma_{\hat{4}\hat{1}} + Y_2\Gamma_{\hat{6}\hat{1}} + Z_7\Gamma_{\hat{3}\hat{2}} + Z_8\Gamma_{\hat{5}\hat{2}}\,, \tag{3.24}$$

$$h_1 = V_1\Gamma_{\hat{4}\hat{1}} + V_2\Gamma_{\hat{6}\hat{1}} + W_1\Gamma_{\hat{3}\hat{2}} + W_2\Gamma_{\hat{4}\hat{2}} + W_3\Gamma_{\hat{5}\hat{2}} + W_4\Gamma_{\hat{6}\hat{2}}\,, \tag{3.25}$$

$$h_2 = V_1\Gamma_{\hat{3}\hat{1}} + V_2\Gamma_{\hat{5}\hat{1}} + W_5\Gamma_{\hat{3}\hat{2}} + W_6\Gamma_{\hat{4}\hat{2}} + W_7\Gamma_{\hat{5}\hat{2}} + W_8\Gamma_{\hat{6}\hat{2}}\,, \tag{3.26}$$

and X,Y,Z,V,W's are defined as

$$
\begin{aligned}
X_1 &= \frac{m_6 - n_4 + (n_6 - m_4)\cos\alpha + n_5 \sin\alpha}{8(C-D)}\,, \qquad X_2 = \frac{n_6 \sin\alpha}{4\sqrt{C^2 - D^2}}\,,\\
X_3 &= \frac{m_6 + n_4 - (m_4 + n_6)\cos\alpha + n_5 \sin\alpha}{8\sqrt{C^2 - D^2}}\,,\\
X_4 &= \frac{m_6 + n_4 + (m_4 + n_6)\cos\alpha - n_5 \sin\alpha}{8\sqrt{C^2 - D^2}}\,,\\
X_5 &= \frac{n_5 \cos\alpha + m_4 \sin\alpha}{4\sqrt{C^2 - D^2}}\,, \qquad X_6 = \frac{-m_6 + n_4 + (n_6 - m_4)\cos\alpha + n_5 \sin\alpha}{8(C+D)}\,,\\
Y_1 &= -\frac{(C' - D')\sin\alpha}{\sqrt{C-D}}\,, \qquad Y_2 = \frac{(C' + D')\sin\alpha}{\sqrt{C+D}}\,,\\
Y_3 &= \frac{(C' - D')\cos\alpha}{\sqrt{C-D}}\,, \qquad Y_4 = -\frac{(C' + D')\cos\alpha}{\sqrt{C+D}}\,,\\
Z_1 &= \frac{n_5 - n_4 \sin\alpha}{2\sqrt{C-D}}\,, \qquad Z_2 = \frac{n_5 + n_4 \sin\alpha}{2\sqrt{C+D}}\,,\\
Z_3 &= \frac{n_6 - n_4 \cos\alpha}{2\sqrt{C-D}}\,, \qquad Z_4 = \frac{n_6 + n_4 \cos\alpha}{2\sqrt{C+D}}\,,\\
Z_5 &= -\frac{m_4 + n_4 \cos\alpha}{2\sqrt{C-D}}\,, \qquad Z_6 = -\frac{m_4 - n_4 \cos\alpha}{2\sqrt{C+D}}\,,\\
Z_7 &= \frac{n_4 \sin\alpha}{2\sqrt{C-D}}\,, \qquad Z_8 = -\frac{n_4 \sin\alpha}{2\sqrt{C+D}}\,,\\
V_1 &= \frac{C' - D'}{\sqrt{C-D}}\,, \qquad V_2 = \frac{C' + D'}{\sqrt{C+D}}\,,\\
W_1 &= \frac{m_6 - m_4 \cos\alpha + n_5 \sin\alpha}{2\sqrt{C-D}}\,, \qquad W_2 = -\frac{n_5 \cos\alpha + m_4 \sin\alpha}{2\sqrt{C-D}}\,,\\
W_3 &= \frac{m_6 + m_4 \cos\alpha - n_5 \sin\alpha}{2\sqrt{C+D}}\,, \qquad W_4 = \frac{n_5 \cos\alpha + m_4 \sin\alpha}{2\sqrt{C+D}}\,,\\
W_5 &= \frac{n_6 \sin\alpha}{2\sqrt{C-D}}\,, \qquad W_6 = -\frac{m_6 + n_6 \cos\alpha}{2\sqrt{C-D}}\,,\\
W_7 &= -\frac{n_6 \sin\alpha}{2\sqrt{C+D}}\,, \qquad W_8 = -\frac{m_6 - n_6 \cos\alpha}{2\sqrt{C+D}}\,.
\end{aligned}
$$

From eq. (3.14), η must be a constant spinor. When $D = 0$, $n_5 = n_6 = m_4 = 0$, $n \equiv n_4 = m_6$ and $A = 2C = 1 + n\,|x^1|$, the metric (3.5) becomes the T-dual of the usual intersecting n NS5-NS5' branes solution. Indeed 1/4 supersymmetries are preserved. For such solution, for example, eq. (3.20) becomes

$$
\begin{aligned}
f &= \frac{n}{2A}\left(2\Gamma_{\hat{1}\hat{2}} - \Gamma_{\hat{4}\hat{5}} + \Gamma_{\hat{3}\hat{6}}\right)\\
&= \frac{n}{2A}\Gamma_{\hat{1}\hat{2}}\left((1 + \Gamma_{\hat{1}\hat{2}\hat{4}\hat{5}}) + (1 - \Gamma_{\hat{1}\hat{2}\hat{3}\hat{6}})\right). \qquad (3.27)
\end{aligned}
$$

So the solution of the condition (3.15) is given by

$$\eta = \frac{1-\Gamma_{\hat{1}\hat{2}\hat{4}\hat{5}}}{2}\frac{1+\Gamma_{\hat{1}\hat{2}\hat{3}\hat{6}}}{2}\,\tilde{\eta}\,, \tag{3.28}$$

where $\tilde{\eta}$ is a constant spinor. It is easy to check that the solution η also satisfy all other conditions (3.16)–(3.19). So the solution (3.28) has unbroken 1/4 supersymmetries for the intersecting NS5-branes.

The NS5-branes with the diamond must preserve the same supersymmetries as eq. (3.28) of intersecting NS5-branes since they are described by the same holomorphic coordinates [21]. The conditions for preserving supersymmetry are

$$\begin{aligned}
\frac{A'}{A} &= 2\,X_3 = 2\,X_4\,, & X_1 &= X_2 = X_5 = X_6 = 0\,,\\
Y_1 &= -Z_2 = Z_8\,, & Y_2 &= -Z_1 = Z_7\,,\\
Y_3 &= Z_4 = Z_6\,, & Y_4 &= Z_3 = Z_5\,,\\
V_1 &= W_3 = -W_8\,, & V_2 &= W_1 = -W_6\,,\\
W_2 &= W_4 = W_5 = W_7 = 0\,.
\end{aligned} \tag{3.29}$$

First, we find $n_4 = m_6$ from $X_1 = X_6 = 0$, then define $n \equiv n_4 = m_6$. Those equations are solved for

$$\begin{aligned}
A = 2\,C &= H\,,\\
2\,D &= \beta\,H\,,\\
n_5 = n_6 &= m_4 = 0\,,
\end{aligned} \tag{3.30}$$

where

$$H \equiv 1 + \frac{n}{\sqrt{1-\beta^2}}\,|x^1|\,. \tag{3.31}$$

Here β is one of integral constants and others are fixed to determine asymptotic behavior of H. From the condition that both $C+D$ and $C-D$ are positive, the range of β is restricted within

$$|\beta| < 1\,. \tag{3.32}$$

When β vanishes, the solution smoothly reduce to the T-dual of intersecting NS5-branes.

Thus we found the solution preserving 1/4 supersymmetries. The metric for the compactified space is

$$ds^2 = H^2\,(dx^1)^2 + H^{-2}\,ds_1^2 + H\,(ds_2^2 + \beta\,ds_3^2)\,, \tag{3.33}$$

where

$$\begin{aligned}
ds_1^2 &= (dx^2 - n\,x^4\,dx^3 - n\,x^6\,dx^5)^2\,,\\
ds_2^2 &= \sum_{i=3}^{6}(dx^i)^2\,,\\
ds_3^2 &= 2\left(\sin\alpha\,(dx^3\,dx^6 + dx^4\,dx^5) + \cos\alpha(dx^4\,dx^6 - dx^3\,dx^5)\right)\,,\\
H &= 1 + \frac{n}{\sqrt{1-\beta^2}}\,|x^1|\,, \qquad |\beta| < 1\,.
\end{aligned} \tag{3.34}$$

The metric (3.33) is a non-compact CY metric. It is straightforward to check that the metric is indeed Ricci flat.

The determinant of the metric (3.33) is proportional to $(1-\beta^2)^2$ which vanishes if $\beta^2 \to 1$. It seems to be the same as the deformation of conifold case where the determinant of the metric vanishes on the S^3 surface. However, since the metric (3.33) is smeared, the meaning of this limit is not so clear. We will discuss the meanings soon later.

Let us consider the compactification of the 10-dimensional type II theory on the 6-dimensional space (3.33),[1] and take the T-duality along the x^2-direction. The duality relations are proposed in [20]. We have the following components relative to the metric (3.33):

$$\begin{aligned} j_{mn} &= g_{mn} - \frac{g_{2m}\, g_{2n}}{g_{22}}\,, \qquad & j_{22} &= \frac{1}{g_{22}}\,, \\ B_{2m} &= -\frac{g_{2m}}{g_{22}}\,, & e^{2\phi} &= \frac{1}{g_{22}}\,. \end{aligned} \tag{3.35}$$

Here m, n are $0, 1, 3, \ldots, 9$, and ϕ is the dilation of the theory after the T-duality. The metric becomes

$$ds^2 = -(dx^0)^2 + H^2 \sum_{i=1}^{2}(dx^i)^2 + H\left(\sum_{i=3}^{6}(dx^i)^2 + \beta\, ds_3^2\right) + \sum_{i=7}^{9}(dx^i)^2\,, \tag{3.36}$$

where

$$ds_3^2 = 2\left(\sin\alpha\,(dx^3\,dx^6 + dx^4\,dx^5) + \cos\alpha(dx^4\,dx^6 - dx^3\,dx^5)\right)\,, \tag{3.37}$$

and the dilaton is given by

$$e^{2\,\phi} = H^2\,. \tag{3.38}$$

The non-vanishing components of the NS-NS 3-form field strength become

$$H_{234} = H_{256} = n\,. \tag{3.39}$$

The metric (3.36) reduces to the intersecting NS5-NS5'-branes metric smoothly if $\beta = 0$, and the preserved supersymmetries are the same as intersecting ones. Therefore we found the metric of the NS5-branes with a diamond smeared except for the x^1-direction and the parameter β corresponds to the size of the diamond.[2]

Let us consider the large size of the diamond $\beta^2 \to 1$. Then the determinant of the metric vanishes and the metric is singular. This is because we have made the metric with the ansatz that the charges of NS5-branes are measured outside the diamond. The ansatz is broken if the point where the charges are measured meets with the diamond. This is a nice correspondence with the deformed conifold metric whose determinant vanishes if one goes on the S^3 surface, $\tau \to 0$ as we have seen in section 2.2.

[1] We may consider both the IIA and IIB theory, since there are no R-R fields.

[2] More precisely $\sqrt{|\beta|}$ can be identified with the size of the diamond, as we will see in section 4.2.

The metric (3.36) describes the NS5-branes with the diamond which is in the delocalized directions. Let us see that the metric (3.36) can transform to the ordinary intersecting one and the diamond shrinks.

First, the parameter α is an angle of a coordinate rotation on either the (x^3, x^4) or (x^5, x^6) plane. If we choose the (x^3, x^4) plane,[3] we can remove it by the following rotation of the coordinates:

$$\begin{aligned} x^3 &\to \cos\alpha\, x^3 + \sin\alpha\, x^4 \,, \\ x^4 &\to -\sin\alpha\, x^3 + \cos\alpha\, x^4 \,. \end{aligned} \tag{3.40}$$

The 3-form field strength (3.39) does not change by the rotation. The off-diagonal part in the metric (3.37) becomes

$$ds_3^2 \to 2\left(dx^4\, dx^6 - dx^3\, dx^5\right), \tag{3.41}$$

and other parts do not change. Thus the parameter α is removed from the solution of NS5-branes with the diamond.

The metric can be diagonalized by the following coordinate transformation:

$$x^3 \to x^3 + \beta\, x^5 \,, \tag{3.42}$$
$$x^6 \to x^6 - \beta\, x^4 \,. \tag{3.43}$$

The terms which associates with the (x^3, x^4, x^5, x^6) directions in the metric become

$$ds_{3456}^2 \equiv H\left((dx^3)^2 + (1-\beta^2)\left((dx^4)^2 + (dx^5)^2\right) + (dx^6)^2\right). \tag{3.44}$$

The components of field strength do not change by the transformation (3.42) and (3.43). Finally we rescale the (x^4, x^5) coordinates as

$$\begin{aligned} x^4 &\to \frac{x^4}{\sqrt{1-\beta^2}} \,, \\ x^5 &\to \frac{x^5}{\sqrt{1-\beta^2}} \,, \end{aligned} \tag{3.45}$$

and the parameter n as

$$n \to \sqrt{1-\beta^2}\, n \,. \tag{3.46}$$

Then we have the intersecting NS5 solution

$$ds^2 = -(dx^0)^2 + H^2 \sum_{i=1}^{2} (dx^i)^2 + H \sum_{i=3}^{6} (dx^i)^2 + \sum_{i=7}^{9} (dx^i)^2 \,, \tag{3.47}$$

where

$$H = 1 + n\, |x^1| \,. \tag{3.48}$$

[3]Even if we choose the (x^5, x^6) plane, the following discussion is appropriate to the plane.

The components of 3-form field strength are the same as (3.39) due to the cancelation of the rescale (3.45) and (3.46). Thus the metric (3.36) reduce to the intersecting one and the diamond shrinks.

We can see the effect of the transformation by using the duality relation [20]. Since we apply it along the transformed coordinates, the transformation correspond to changing the direction of the compactification.

4. U-dualities

The (deformed) conifold geometry or NS5-branes configurations are related to other interesting configurations by dualities. In this section we consider the consistency under the dualities.

4.1 NS5-brane and D4-brane

We consider the type IIB theory on the conifold. The T-duality along the x^2 direction take us the configuration of intersecting NS5-NS5' branes in the type IIA theory whose world-volume directions are

$$\begin{array}{r|l} \mathrm{NS5} & (0 \quad 3\ 4 \qquad 7\ 8\ \ 9) \\ \mathrm{NS5}' & (0 \qquad\quad 5\ 6\ 7\ 8\ 9)\,. \end{array} \tag{4.1}$$

We first lift the configuration to the M-theory and flip the x^5 and x^{10} directions.

$$\begin{array}{r|l} M5 & (0 \quad 3\ 4 \qquad 7\ 8\ 9 \quad\ \ ) \\ M5' & (0 \qquad\quad 6\ 7\ 8\ 9\ 10)\,. \end{array} \tag{4.2}$$

We dimensionally reduce the x^{10} direction and have the intersecting NS5 and D4-branes whose world-volume directions are

$$\begin{array}{r|l} \mathrm{NS5} & (0 \quad 3\ 4 \qquad 7\ 8\ 9) \\ D4 & (0 \qquad\quad 6\ 7\ 8\ 9) \end{array} \tag{4.3}$$

in the type IIA theory since one of M5-branes wraps on the compactified circle.

The D4-brane ends on the NS5-brane and each open D4-branes can slide along the NS5-brane [22]. The shift of the open D4-branes in the (x^3, x^4) space corresponds to the size of the diamond. This can be seen as follows. We introduce the holomorphic coordinate $x = x^3 + i\,x^4$. Let the position of the one of the open D4-branes on the x plane be $x = 0$ and the other be $x = m$. In M-theory, these branes are described by the M5-brane wrapping on the holomorphic curve [9]

$$x\,t - (x - m) = 0\,, \tag{4.4}$$

where $t = e^y$, $y = x^6 + ix^{10}$ and we take the radius of the x^{10}-direction as 1. We choose coefficients of the curve (4.4) as that the solutions of the curve are $x = 0, y = 0$ when $m = 0$. In the near end points of D4-branes limit $|y| \ll 1$, since $t \sim 1 + y$, the curve (4.4) becomes the same as the curve (2.5). Therefore we find that the squareroot of $|m|$ corresponds to the size of the diamond ϵ.

This correspondence is generalized to intersecting n NS5-branes and m NS5'-branes in IIA theory [11]. In this case, we have a generalized conifold ($\mathcal{G}_{mn} : uv = x^m y^n$) after the T-duality along the overall transverse direction to the NS5 and NS5'-branes. The generalized conifold can be deformed to a smooth space by

$$uv = \sum_{i,j=0}^{n,m} m_{ij} x^i\, y^j\,. \tag{4.5}$$

So, after the T-duality, we have the curve

$$\begin{aligned} 0 &= \sum_{i,j=0}^{n,m} m_{ij} x^i\, y^j \\ &= (m_{m\,n} x^m + \cdots + m_{0\,n})\, y^n + (m_{m\,n-1} x^m + \cdots + m_{0\,n-1})\, y^{n-1} + \cdots \\ &\quad + (m_{m\,0} x^m + \cdots + m_{0\,0}) \end{aligned} \tag{4.6}$$

on which a single NS5-brane wraps. The parameters m_{ij} correspond to the locations of the original NS5 and NS5'-branes and the size of diamonds opened at each intersecting points. If we lift to the M-theory, we have a single M5-brane wrapping the Seiberg-Witten curve which is the same as the curve (4.6). We have an $N = 2$ four-dimensional $\mathrm{SU}(m)^{n-1}$ gauge theory with vanishing beta functions on D4-branes after the coordinate flip and the dimensional reduction along the direction in originally the NS5'-branes. The matters consist of the $(n-2)$ hypermultiplets in the bi-fundamental representation and two hypermultiplets in the fundamental representation of $\mathrm{SU}(m)$, which come from the semi-infinite D4-branes in the left and right. Thus the size of diamonds or locations of NS5 and NS5'-branes corresponds to the moduli parameters of the $N = 2$ gauge theory. In fact the $(m-1)+(n-1)$ relative branes positions and the size of diamonds at mn intersection are mapped to $(n-2)+2m$ bare masses of hypermultiplets, $(n-1)(m-1)$ vevs of scalars in adjoint representation of the gauge groups $\mathrm{SU}(m)^{n-1}$ and $n-1$ complex gauge coupling constants. Since all beta-functions are zero, gauge coupling constants are also moduli parameters. As a result, we have $mn+m+n-2$ parameters in total. This number exactly agrees with the deformation parameter in eq. (4.5).

4.2 Other U-dualities

Let us consider the correspondence between the displacement of the ends of D4-branes and the size of the diamond by using the NS5-brane with the diamond metric (3.36) .

We consider the following duality maps. First, we start with the configuration of intersecting NS5 and D4-branes,

$$\begin{array}{r|l} \mathrm{NS5} & (\,0 \quad 3\;4 \qquad 7\;8\;9\;) \\ D4 & (\,0 \qquad\qquad 6\;7\;8\;9\;)\,. \end{array} \tag{4.7}$$

Secondly, the T-duality along the x^7, x^8 and x^9-directions maps the configuration to

$$\begin{array}{r|l} \mathrm{NS5} & (\ 0 \quad 3\ 4 \quad\ \ 7\ 8\ 9\ ) \\ D1 & (\ 0 \qquad\quad 6 \qquad\quad)\,. \end{array} \tag{4.8}$$

Finally, we apply S-dual operation and and obtain

$$\begin{array}{r|l} D5 & (\ 0 \quad 3\ 4 \quad\ \ 7\ 8\ 9\ ) \\ F1 & (\ 0 \qquad\quad 6 \qquad\quad)\,. \end{array} \tag{4.9}$$

It is shown in [23] that if there is the displacement of the two ends of the fundamental string along the x^i-direction on the D-brane, the displacement becomes a constant B-field after the T-duality along the direction. The relation of the displacement δx^i and the component of the B-field is given by

$$\delta x^i = B_{i6}\,, \tag{4.10}$$

where the fundamental string extends along the x^6-direction. Therefore if we take the magnitude of the displacement along the x^3, x^4-directions as $\delta x^3, \delta x^4$ respectively, we have components of the B-field such that

$$\begin{aligned} B_{36} &= \delta x^3 \\ B_{46} &= \delta x^4 \end{aligned} \tag{4.11}$$

after the T-duality along the x^3 and x^4-directions.

It is apparent that the displacement of the end points of D4-branes on the (x^3, x^4) directions is the same as that of fundamental strings in the directions under the duality map from the configuration (4.7) to (4.9). The diamond size corresponds to the B-field since the displacement $\delta x = m$ corresponds to it as we discussed in the previous subsection. We confirm the observation by using the NS5-branes with a diamond metric (3.36) and the duality relations given in [20]. We can also identify with the relation between the parameter β and the size of the diamond.

We trace the duality chain (4.1)–(4.3), (4.8) and (4.9) and take the T-dualities to the configuration (4.9) along the x^3, x^4 direction. Applying the dualities relations to the metric (3.36), we obtain

$$\begin{aligned} B_{36} &= \beta\,\cos\alpha\,, \\ B_{46} &= -\beta\,\sin\alpha\,. \end{aligned} \tag{4.12}$$

From (4.11) we have

$$\delta x\,(=m) = \beta\,\cos\alpha - i\,\beta\,\sin\alpha\,. \tag{4.13}$$

Therefore we find that $\sqrt{|m|} = \sqrt{|\beta|}$ is the size of the diamond. The direction of the displacement is rotated in the (x^3, x^4) directions at the angle α. We confirm the discussion in the section 3.

5. Conclusion and discussion

We start with the metric inspired by the deformed conifold metric, and obtain the NS5-branes with the diamond metric which is smeared except for one of the overall transverse directions. The metric is also obtained from the intersecting NS5-branes one by the transformation (3.42) and (3.43). It means that the diamond is spread by the transformation. The parameter β in the metric is the square of the size of the diamond and α is the rotation angle on the plane over which originally one of intersecting NS5-branes extends. The NS5-brane with the diamond relates to the NS5-brane and the D4-brane via string dualities, where the D4-brane breaks on the NS5-brane. The displacement of the end points is equal to β.

The metric of NS5-branes with the diamond is delocalized and does not have local information about the diamond. However we can get the information about the size of the diamond because we assume that there is only one diamond. If there are more than one diamond as in section 4.1, it is difficult to obtain information about each diamonds from smeared metrics.

The partially localized solutions for the intersecting branes are obtained [24, 25, 26, 27]. The fully localized solution for a M5-brane wrapping a Riemann surface is also discussed in [25]. The fully localized solution is presented with a Kähler potential, however, the explicit form of the Kähler potential is not known. In [28], the authors discuss the Kähler potential perturbatively for various types of intersecting branes and branes wrapping curves, and find that the perturbation theory is well behaved at least to the second order if there are less than three overall transverse dimensions.

It is an interesting problem how the localized intersecting branes solutions can be deformed and how the conifold or the deformed conifold relate to such solutions by the duality map.

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